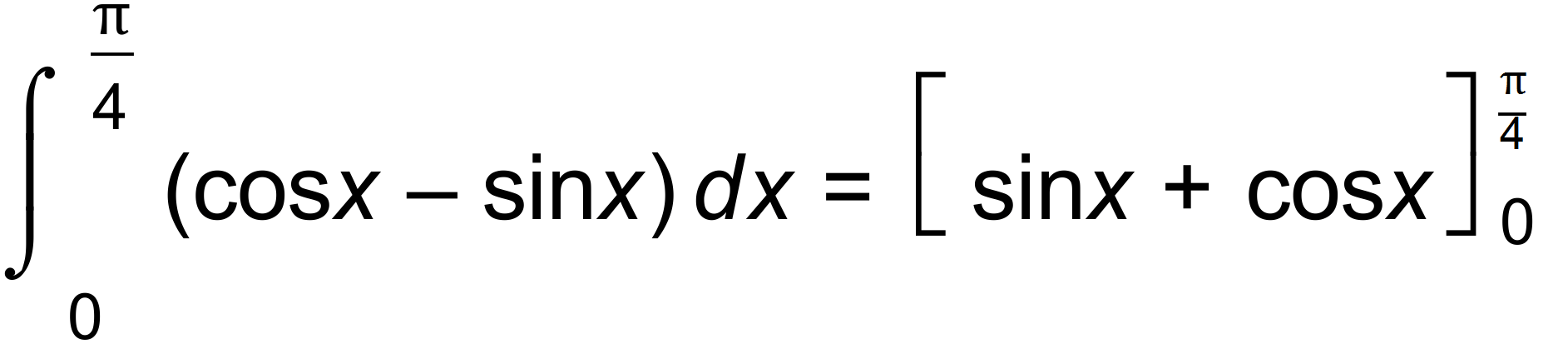
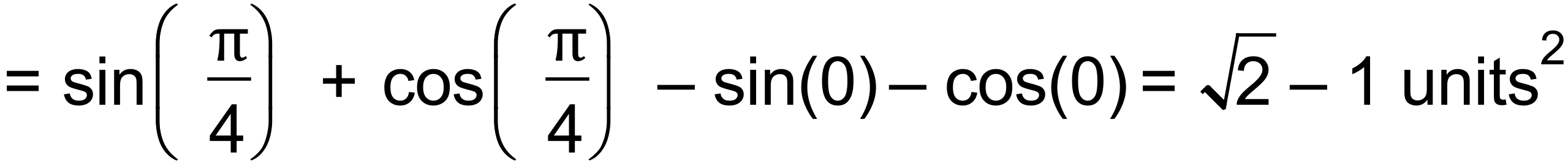
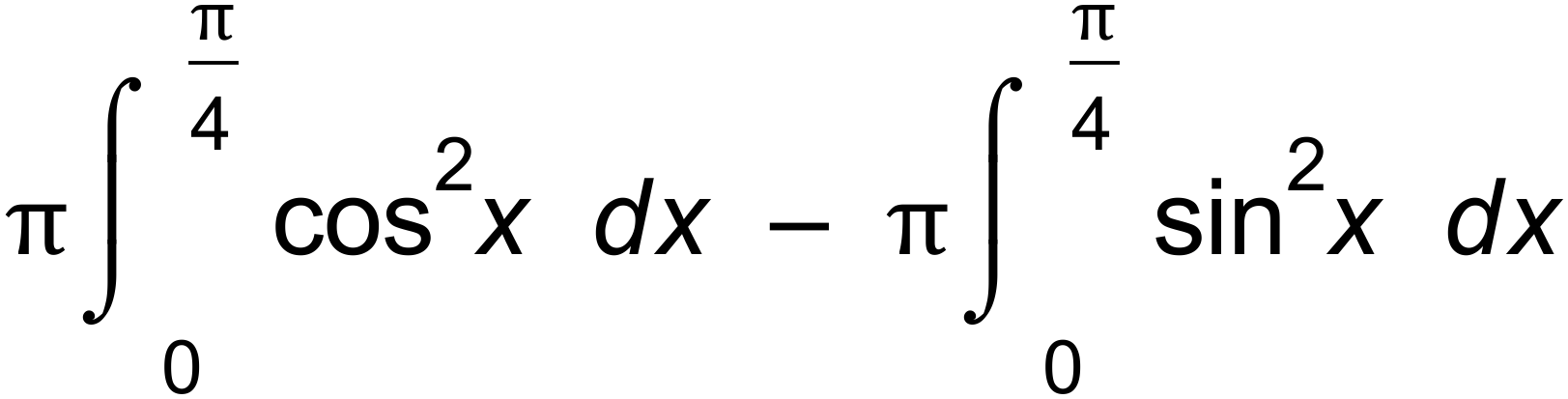
|  |  |
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|  | **MATHEMATICS:SPECIALIST 3 & 4**  **SEMESTER 2 2017**  **TEST 4**  **Calculator Free** |
|  |  |

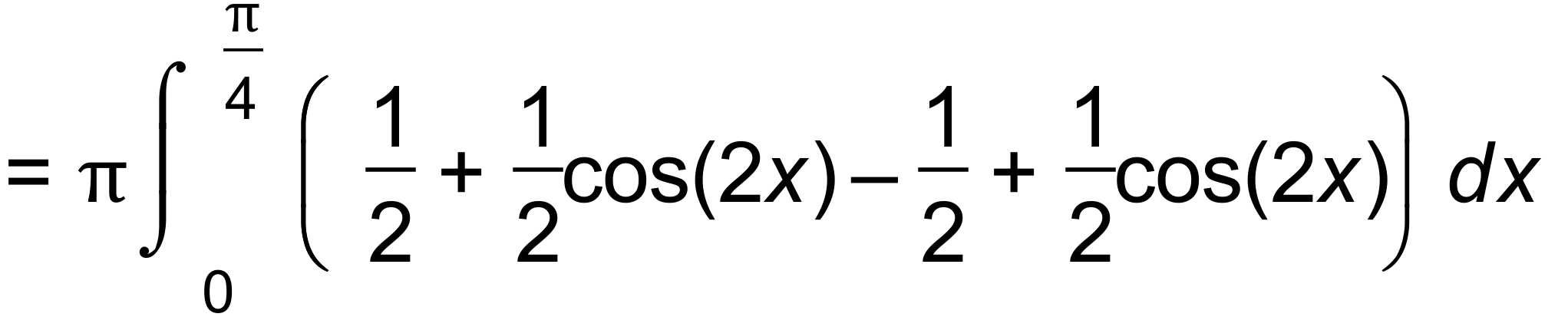
Reading Time: 2 minutes Time Allowed: 25 minutes

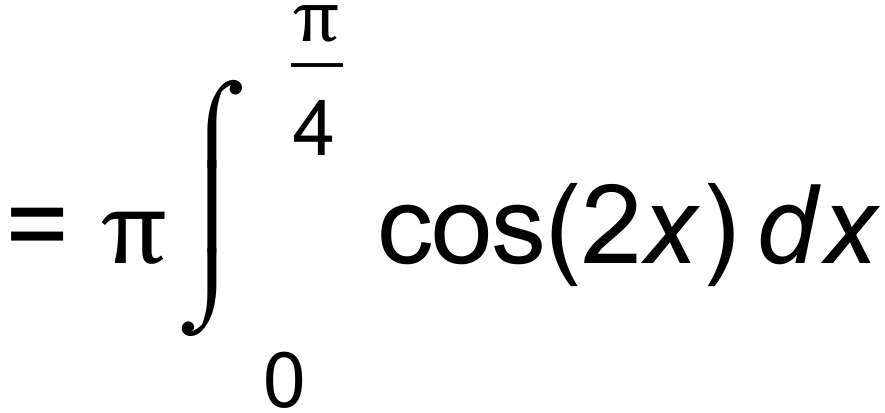
Total Marks: 22

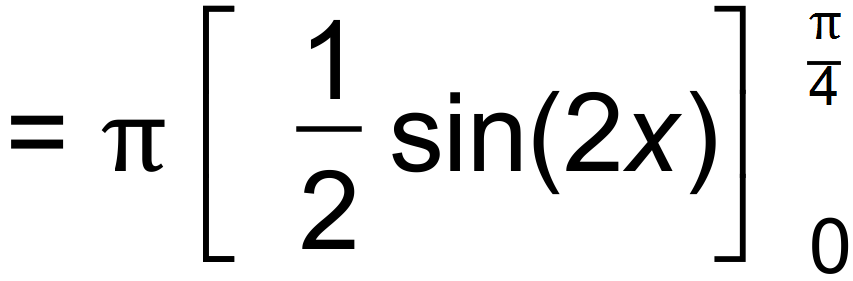
1. (a)  ✓

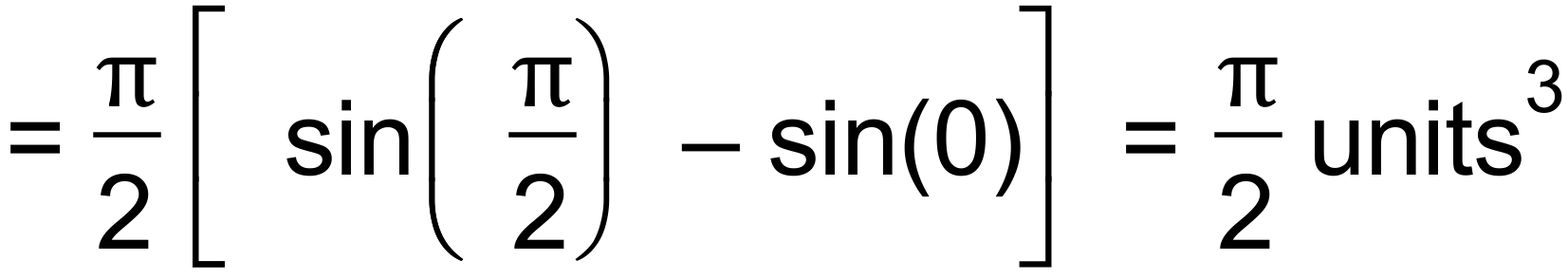
 ✓

(b)  ✓

 ✓

 ✓

 ✓

 ✓ [7]

**Question 2 (15 marks)**

Determine the following integrals.

1.  (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 recognises that numerator is proportional to derivative of numerator  🗸 uses natural log with a constant |

1.  (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 uses double angle formula for cosine  🗸 integrates one term correctly  🗸 integrates all terms correctly |

Determine the following integrals with the given substitution.

1.  (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 changes variable to *u* in integral  🗸 antidifferentiates with respect to *u*  🗸 expresses in terms of *x* |

1.  (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 changes variable to *u* in integral  🗸 changes limits to *u* values  🗸 determines definite integral |

(e) Using partial fractions, or otherwise, determine . (4 marks)

|  |
| --- |
| **Solution** |
| Solving gives |
| **Specific behaviours** |
| ✓ writes equations for A and B  ✓ determines A and B  ✓ integrates both fractions correctly  ✓ includes constant of integration |

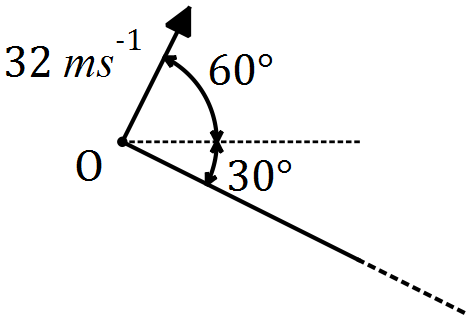
|  |  |
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|  | **MATHEMATICS:SPECIALIST 3 & 4**  **SEMESTER 2 2017**  **TEST 4**  **Calculator Assumed** |
|  |  |

Reading Time: 2 minutes Time Allowed: 30 minutes

Total Marks: 28

Question 3 (12 marks)

A small body is projected upwards from the top of a hill with an initial velocity of 32 ms-1 at an angle of 60° to the horizontal. The hill slopes downwards at a constant angle of 30° to the horizontal. Let the origin of a cartesian coordinate system be the top of the hill, with a unit vector in the positive direction and a unit vector in the positive direction. Displacement is measured in metres and time in seconds.



(a) Show that the initial velocity of the body is . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses trig ratios |

The acceleration of the body, seconds after projection, is given by **.**

(b) Determine an expression for the position vector of the body after seconds. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ integrates acceleration  ✓ uses initial velocity for constant  ✓ integrates velocity |

(c) Determine the time at which the body lands on the hillside. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses ratio of coefficients and tangent of slope  ✓ writes equation using position coefficients  ✓ solves equation |

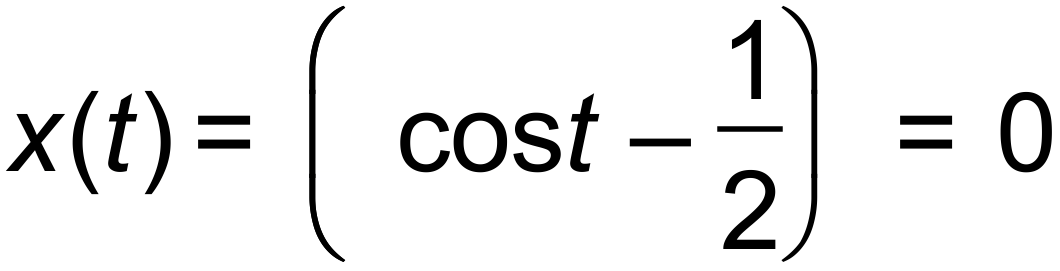
(d) Calculate the distance of the body from at the instant it lands. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines position  ✓ calculates magnitude |

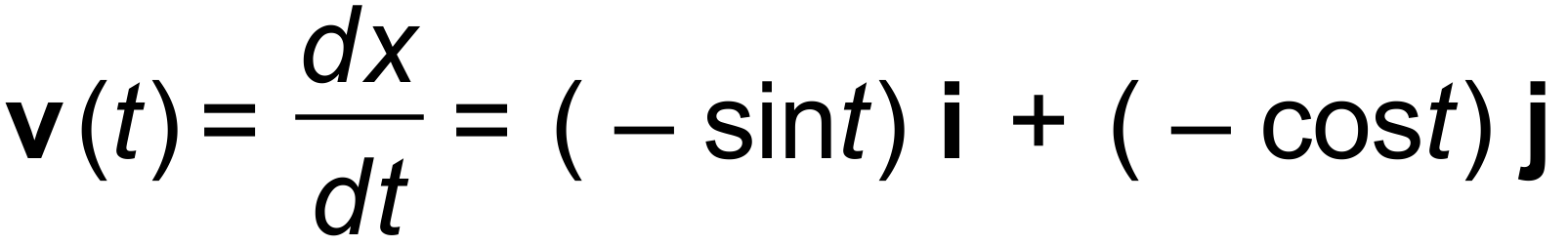
(e) Determine the maximum vertical height attained by the particle above the hillside.

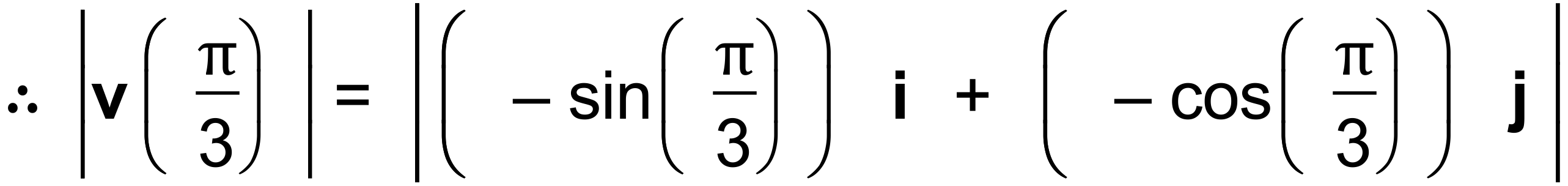
(3 marks)

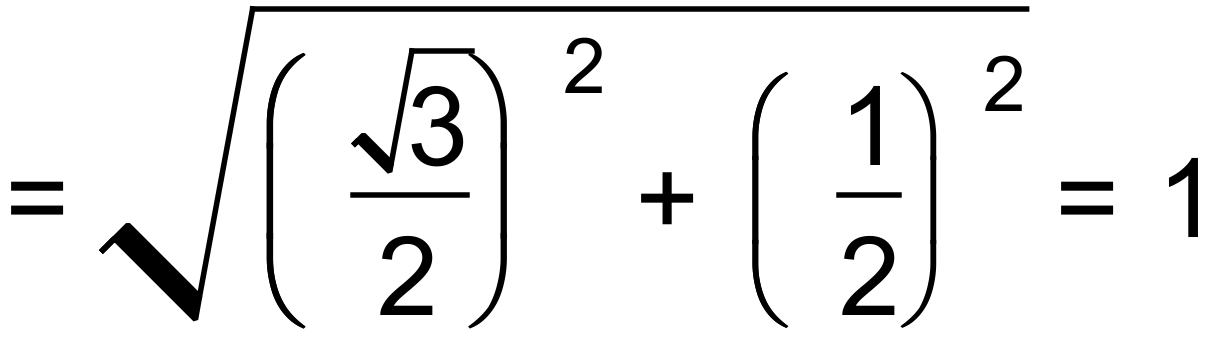
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ forms equation for height above hillside  ✓ solves equation for maximum  ✓ determines height |

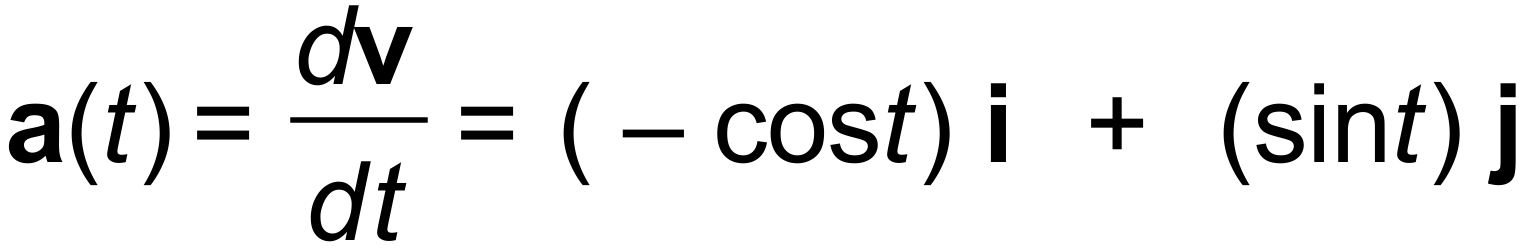
4. (a) 

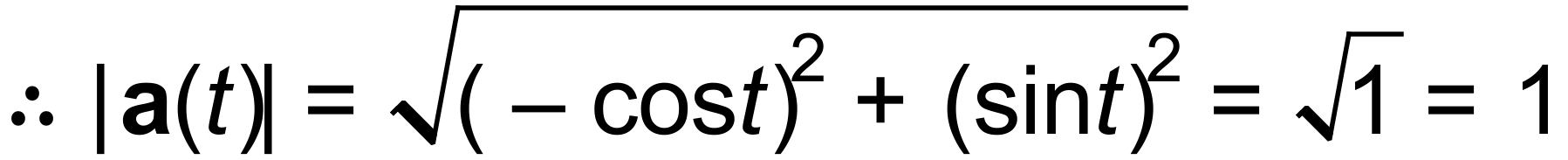
 ✓

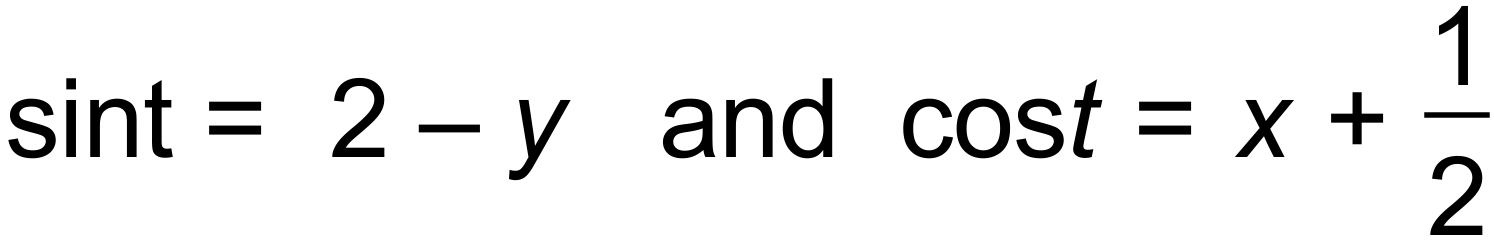
 ✓

 ✓

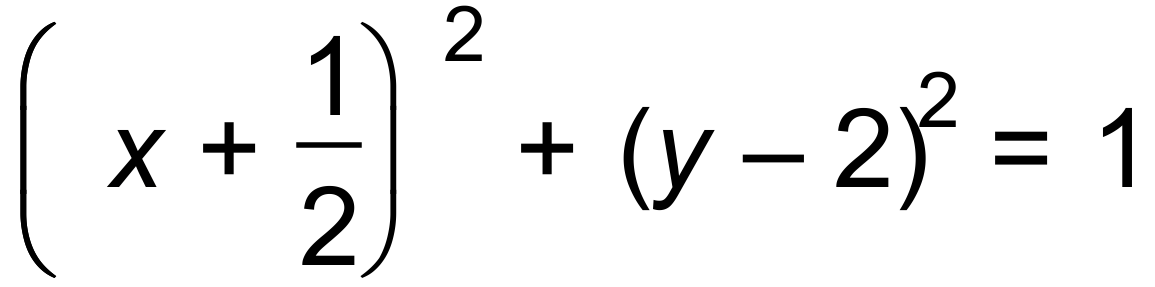
 units/s ✓

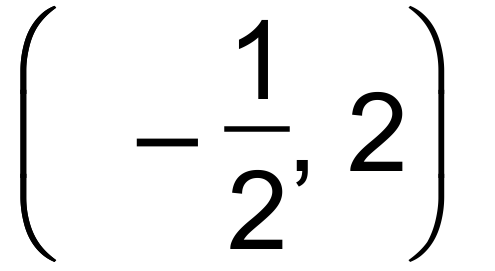
(b)  ✓

 unit/s2 ✓

(c) 

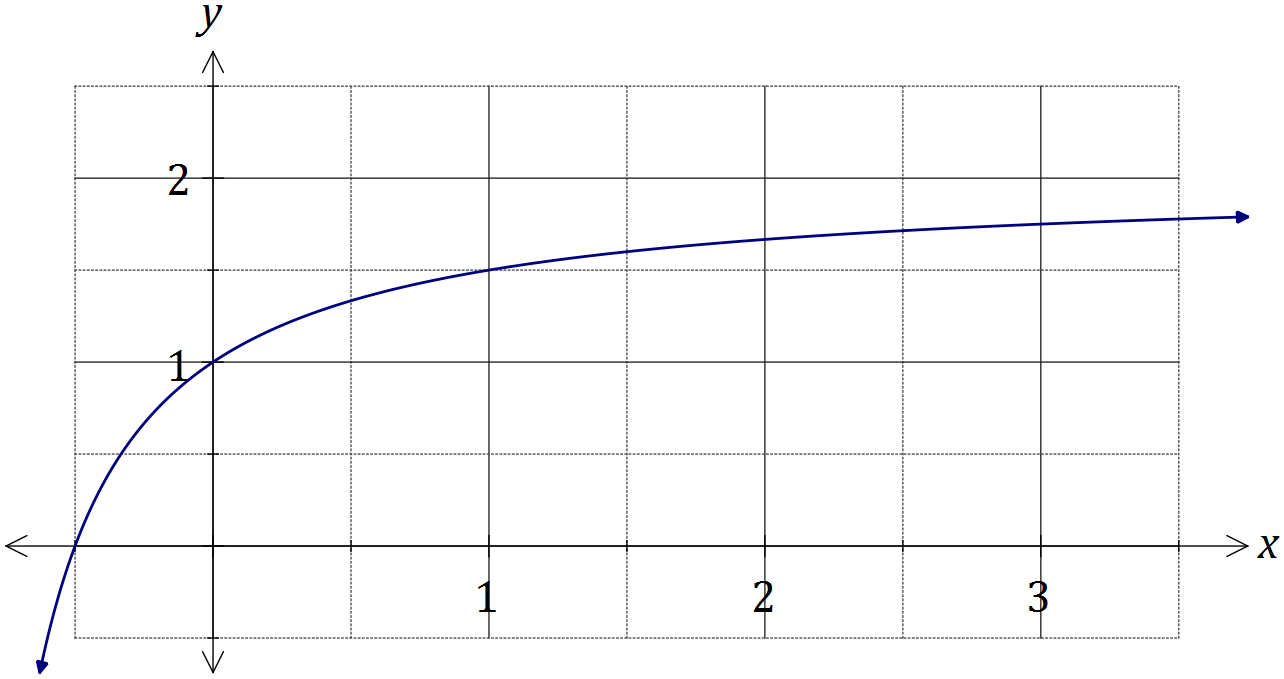
since 

then  ✓

the path is a circle centred at  of radius = 1 unit ✓ [8]

Question 5 (8 marks)

(a) Sketch the graph of on the axes below. (2 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ axes intercepts and (1, 1)  ✓ smooth curve |

Simpson's rule is a formula used for numerical integration, the numerical approximation of definite integrals. When an interval is divided into an even number, , of smaller intervals of equal width , the bounds of these smaller intervals are denoted . Simpson's rule can be expressed as follows:

where , is the sum of the values of where is even and is the sum of the values of where is odd.

(b) Use Simpson's rule with to evaluate an approximation for , correct to four decimal places. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates ordinates  ✓ calculates B, E and O  ✓ evaluates using rule  ✓ accurate to 4 dp |

(c) Determine the exact value of and hence calculate the percentage error of the approximation from (b). (2 marks)

|  |
| --- |
| **Solution** |
| . Hence error. |
| **Specific behaviours** |
| ✓ calculates exact value  ✓ calculates % error |